Department of Education

First Steps in Mathematics

Overview

Improving the mathematics outcomes of students

First steps in Mathematics: Overview

Revised edition

ISBN: 978-0-7307-4482-5
SCIS: 1593589

Acknowledgments
The authors and publishers would like to thank the following people and organisations for granting their permission to reproduce copyright material.

For extracts: Preston, B., 1993, 'Teacher professionalism: Implications for teachers, teacher educators and democratic schooling', *Independent Education*, 23, 4, 4-12, pp. 2, 20; Darling-Hammond, L., 1994, 'National standards and assessments: Will they improve education?', *American Journal of Education*, 102, 478-51, p. 4.

For illustrations: Ian Forss

For photographs: Lindsay Edwards Photography

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Foreword

The ***First Steps in Mathematics*** Resource Books and professional development program are designed to help teachers to plan, implement and evaluate the mathematics curriculum they provide for their students. The series describes the key mathematical ideas students need to understand in order to achieve the mathematics outcomes described in the Western Australian *Curriculum Framework* (1998).

Each Resource Book is based on five years of research by a team of teachers from the Department of Education and Training, and tertiary consultants led by Professor Sue Willis at Murdoch University. The ***First Steps in Mathematics*** project team conducted an extensive review of national and international research literature, which revealed gaps in the field of knowledge about students' learning in mathematics.

Using tasks designed to replicate those in the research literature, team members interviewed students in diverse locations. Analysis of the data obtained from these interviews identified characteristic phases in the development of students' thinking about major mathematical concepts. The Diagnostic Maps—which appear in the Resource Books for Number, Measurement, Space, and Chance and Data—describe these phases of development.

It has never been more important to teach mathematics well. Globalisation and the increasing use of technology have created changing demands for the application of mathematics in all aspects of our lives. Teaching mathematics well to all students requires a high level of understanding of teaching and learning in mathematics and of mathematics itself. The ***First Steps in Mathematics*** series and professional development program will enhance teachers' capacity to decide how best to help all of their students achieve the mathematics outcomes.

The commitment and persistence of many teachers and officers of the Department of Education and Training, who contributed to the research and development of ***First Steps in Mathematics***, is acknowledged and appreciated. Their efforts have resulted in an outstanding resource for teachers. I commend the series to you.

Paul Albert
Director General
Department of Education and Training
Western Australia

CHAPTER 1

What Is *First Steps in Mathematics*?

First Steps in Mathematics is a series of teacher resource books that is organised around sets of mathematics outcomes for Number, Measurement, Space, and Chance and Data. The series will help teachers to diagnose, plan, implement and judge the effectiveness of the learning experiences they provide for students.

The aim of ***First Steps in Mathematics*** is to improve the mathematics outcomes of students. The series and professional development do this by improving teachers' understandings of teaching and learning in mathematics within a developmental framework.

First Steps in Mathematics was developed in response to the Department of Education and Training in Western Australia adopting an outcomes-focused curriculum in the early 1990s. An outcomes-focused curriculum requires teachers to focus on what students have actually learned as a result of their teaching. As work on this initiative progressed, it became apparent that new approaches to professional development and teacher resources would be needed to support primary teachers of mathematics.

The ***First Steps in Mathematics*** authors used extensive research to develop these resources. The research phase of the project coincided with the introduction of the *Curriculum Framework for K–12 Education in Western Australia*, a document that defines the outcomes expected of all students.

Professional Judgment

> *Professionalism has one essential feature; …(it) requires the exercise of complex, high level professional judgments…(which) involve various mixes of specialised knowledge; high level cognitive skills; sensitive and sophisticated personal skills; broad and relevant background and tacit knowledge.*
>
> **Preston, 1993, pp. 2, 20**

First Steps in Mathematics recognises that the professional judgments involved in teaching mathematics well to all students in all circumstances cannot be made simple, technical, or routine. It requires a combination of knowledge, experience and evidence.

First Steps in Mathematics is not meant to replace professional judgment. Rather, it should ensure that teachers' professional judgments about mathematics, assessment, teaching and learning are well informed. The series and associated professional development will help teachers to:

- build or extend their own knowledge of the mathematics underpinning the outcomes
- understand how students learn mathematics so they can make sound professional judgments
- plan learning experiences that are likely to develop the mathematics outcomes for all students.

Teachers will also learn to recognise opportunities for incidental teaching during conversations and routines that occur in the classroom.

CHAPTER 2

Beliefs About Teaching and Learning

First Steps in Mathematics is underpinned, or supported, by a number of beliefs about effective teaching and learning.

The Explicit Statement of Long-Term Mathematics Outcomes Improves Clarity of Focus

Learning is improved if the whole-school community has a shared understanding of the mathematics outcomes, and a commitment to achieving them. A common understanding of these long-term outcomes helps individuals and groups of teachers decide how best to aid students' learning, and how to tell when this has happened.

All Students Can Learn Mathematics to the Best of Their Ability

A commitment to common outcomes signals a belief that all students can be successful learners of mathematics. A situation where less is expected of and achieved by certain groups of students is not acceptable. School systems, schools and teachers are all responsible for ensuring that each student has access to the learning conditions he or she requires to achieve the outcomes to the best of his or her ability.

Learning Mathematics Is an Active and Productive Process

Learning is not simply about the transfer of knowledge from one person to another. Rather, students need to construct their own mathematical knowledge in their own way and at a pace that enables them to make sense of the mathematical situations and ideas they encounter. A developmental learning approach is based on this notion of learning. It recognises that not all students learn in the same way, through the same processes, or at the same rate.

Common Outcomes Do Not Imply Common Curriculum

The explicit statement of the outcomes expected for all students help teachers to make decisions about the curriculum. However, the outcomes are not a curriculum. If all students are to succeed to the best of their ability on commonly agreed outcomes, different curricula will not only be possible, but also be necessary. Teachers must decide what curriculum is needed for their students to achieve the outcomes.

> *A curriculum that enables all students to learn must allow for different starting points and pathways to learning so that students are not left out or behind.*
>
> **Darling-Hammond, 1994, p. 480**

Professional Judgment Is Central in Teaching

It is the responsibility of teachers to provide all students with the conditions necessary for them to achieve the outcomes. This responsibility requires teachers to make continual professional judgments about what to teach, to whom, and how.

The personal nature of each student's learning means that the decisions teachers make are often 'non-routine', and the reasoning processes involved can be complex. These processes cannot be reduced to a set of instructions about what to do in any given situation. Teachers must have the freedom and encouragement to adapt existing curricula flexibly to their students' needs. The improvement of students' learning is most likely to take place when teachers have good information on which to base their professional judgments.

'Risk' Relates to Future Mathematics Learning

Risk cannot always be linked directly to students' current achievement. Rather, it refers to the likelihood that their future mathematical progress is 'at risk'.

Some students who can answer questions correctly might not have the depth of understanding needed for ongoing progress. Others might have misconceptions that could also put their future learning 'at risk'. A number of students may make errors that are common when they try to make sense of new mathematical ideas and, therefore, show progress. For example, a student who writes six hundred and four as '6004' is incorrect. However, this answer signals progress because the student is using his or her knowledge of the fact that the hundreds are written with two zeros.

Students who are learning slowly, or whose previous experiences are atypical, might nevertheless progress steadily if their stage of learning is accommodated with appropriate, but challenging, learning experiences.

Successful Mathematics Learning Is Robust Learning

Robust learning, which focuses on students' achieving the mathematics outcomes fully and in depth, is essential if learning is to be sustained over the long term.

A focus on short-term performance or procedural knowledge at the expense of robust knowledge places students 'at risk' of not continuing to progress throughout the years of schooling.

CHAPTER 3

Learning Mathematics

Learning mathematics is an active and productive process on the part of the learner. The following pages illustrate how this approach influences the ways in which mathematics is taught in the classroom.

Learning Is Built on Existing Knowledge

Learners' interpretations of mathematical experiences depend on what they already know and understand. For example, many young students start school able to count collections of seven or eight objects by pointing and saying the number names in order. However, they may not have the visual memory to recognise seven or eight objects at a glance. Others may readily recognise six or seven objects at a glance without being able to say the number names in order.

In each case, students' existing knowledge should be recognised and used as the basis for further learning. Their learning should be developed to include the complementary knowledge, with the new knowledge being linked to and building on students' existing ideas.

Learning Requires That Existing Ideas Be Challenged

Learning requires that students extend or alter what they know as a result of their knowledge being challenged or stretched in some way. For example, a challenge may occur when a student predicts that the tallest container will hold the most water, then measures and finds that it does not.

Another challenge may occur when a student believes that multiplication makes numbers bigger and then finds that this is not true for some numbers. Or, it may happen when the student finds that peers think about a problem in a different way. The student must find some way of dealing with the challenge or conflict provided by the new information in order to learn.

Learning Occurs when the Learner Makes Sense of the New Ideas

Teaching is important—but learning is done *by* the learner rather than *to* the learner. This means the learner acts on and makes sense of new information. Students almost always try to do this. However, in trying to make sense of their mathematical experiences, some students will draw conclusions that are not quite what their teachers expect.

Also, when students face mathematical situations that are not meaningful, or are well beyond their current experience and reach, they often conclude that the mathematics does not make sense or that they are incapable of making sense of it. This may encourage students to resort to learning strategies based on the rote imitation of procedures. The result is likely to be short-term rather than effective long-term learning. Teachers have to provide learning experiences that are meaningful and challenging, but within the reach of their students.

Learning Involves Taking Risks and Making Errors

In order to learn, students have to be willing to try a new or different way of doing things, and stretch a bit further than they think they can. At times, mistakes can be a sign of progress. For example, students often notice that each number place, from right to left, has a new name (i.e. ones, tens, hundreds and thousands), but they may predict incorrectly that the next place will be millions. Such errors can be a positive sign that students are trying to generalise the patterns in the way we write numbers.

Errors can provide a useful source of feedback, challenging students to adjust their conceptions before trying again. Errors may also suggest that learners are prepared to work on new or difficult problems where increased error is likely. Or, they may try improved ways of doing things that mean giving up old and safe, but limited, strategies. For example, a student who can successfully find 'five twenty sixes' by adding the number 26 five times takes a risk when trying to do it by multiplying, since multiplying may result in increased mistakes in the short term.

Learners Get Better with Practice

Students should get ample opportunity to practise mathematics, but this involves much more than the rote or routine repetition of facts and procedures. For example, if students are to learn how to plan data collection, they will need plenty of opportunities to actually plan their own surveys and experiments, note for themselves when things don't work as expected, and improve their collection processes to improve their data.

Likewise, if students are to develop good mental arithmetic, they will need spaced and varied practice with a repertoire of alternative addition strategies and with choosing among them. Extensive repetitive practice on a single written addition algorithm is unlikely to help with this. In fact, it is more likely to interfere with it.

Learning Is Helped by Clarity of Purpose for Students as well as Teachers

Learning is likely to be enhanced if students understand what kind of learning activity they should be engaged in at any particular time. This means helping students to distinguish between tasks that provide practice of an already learned procedure and tasks that are intended to develop understanding of mathematical concepts and processes. In the former case, little that is new is involved, and tasks are repetitive, so they become habitual and almost unthinking. Students should expect to be able to start almost immediately and, if they cannot, realise that they may need to know more and seek help.

With tasks that are intended to develop understanding, non-routine tasks and new ideas may be involved. Students should not expect to know what to do or to be able to get started immediately.

Students may spend a considerable amount of time on a single task, and they will often be expected to work out for themselves what to do. They should recognise that, for such activities, persistence, thoughtfulness, struggle and reflection are expected.

Teaching Mathematics

Teachers assume considerable responsibility for creating the best possible conditions for learning. The kind of learning tasks and environment teachers provide depends on their own view of how learning is best supported. The perspective that learning is an active and productive process has two significant implications for teaching.

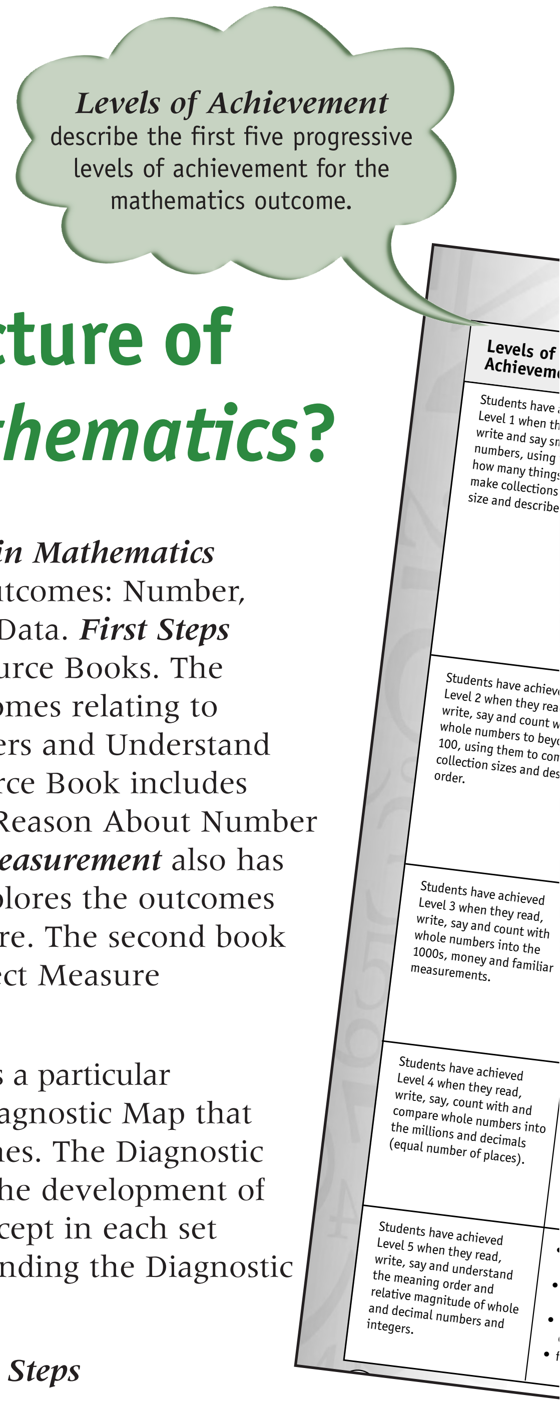
First, teachers cannot predict or control exactly what and when students learn. They need to plan curricula that provide students with a wider and more complex range of information and experiences than they would be expected to understand fully at any given time. For example, using the constant function on their calculators, Year 1 students may be able to 'count' into the thousands. Their teacher may encourage students' exploration, read the numbers for them and stimulate their curiosity about large numbers in general. This enables students to begin developing notions about counting and numbers at many different levels. However, their teacher may only *expect* them to demonstrate a full understanding of the connection between quantity and counting for small numbers.

This represents a significant change in curriculum planning. It is a movement away from an approach that only exposes students to content and ideas that they should be able to understand or do at a particular point in time.

Second, for students to become effective learners of mathematics they must be fully and actively engaged. Students will want, and be able, to take on the challenge, persistent effort and risks involved. Equal opportunities to learn mathematics means teachers will:

- provide an environment for learning that is equally supportive of all students
- offer each student appropriate mathematical challenges
- foster in all students processes that enhance learning.

CHAPTER 4



What Is the Structure of *First Steps in Mathematics*?

Each Resource Book in the ***First Steps in Mathematics*** series focuses on one of these sets of outcomes: Number, Measurement, Space, and Chance and Data. ***First Steps in Mathematics: Number*** has two Resource Books. The first Resource Book examines the outcomes relating to Understand Whole and Decimal Numbers and Understand Fractional Numbers. The second Resource Book includes Calculate, Understand Operations and Reason About Number Patterns. ***First Steps in Mathematics: Measurement*** also has two Resource Books. The first book explores the outcomes for Understand Units and Direct Measure. The second book features the outcomes relating to Indirect Measure and Estimate.

Each Resource Book in the series follows a particular structure, and is to be used with the Diagnostic Map that has been created for each set of outcomes. The Diagnostic Map describes characteristic phases in the development of students' thinking about the major concept in each set of outcomes. (See Chapter 5: 'Understanding the Diagnostic Maps'.)

As well as a Diagnostic Map, each ***First Steps in Mathematics*** Resource Book includes:

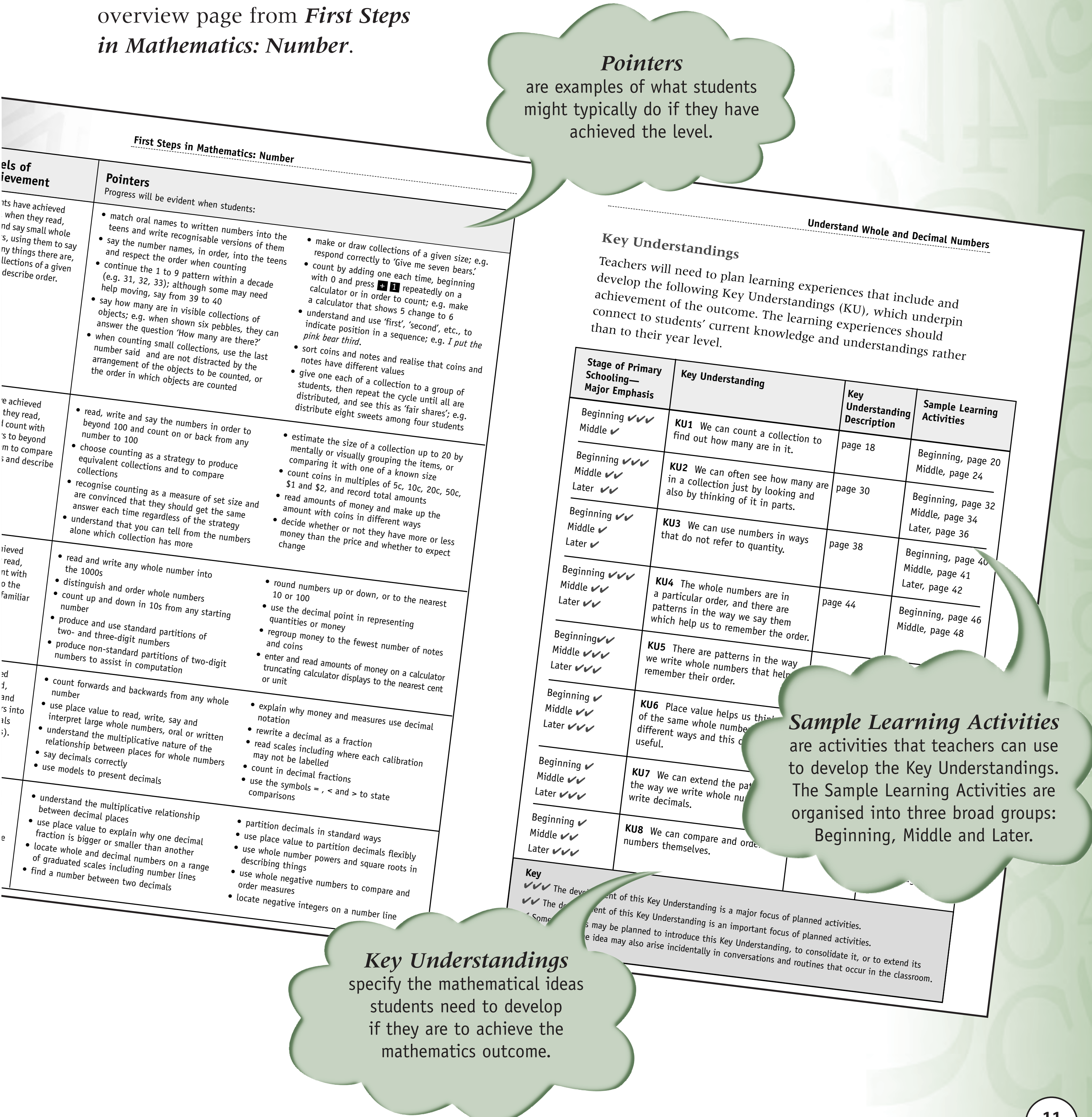
- Sample Learning Activities
- Sample Lessons
- 'Did You Know?' sections.

Each of these elements is described in detail in this chapter.

Also, each outcome features an overview page that highlights:

- Levels of Achievement
- Pointers
- Key Understandings.

The diagram below shows a sample overview page from ***First Steps in Mathematics: Number***.



Mathematics Outcomes

The ***First Steps in Mathematics*** resources and professional development support planning for teaching and learning in relation to the mathematics outcomes from the Western Australian *Curriculum Framework*. The mathematics outcomes indicate what students are expected to know, understand and be able to do as a result of their learning experiences.

There are sets of outcomes for Working Mathematically, Number, Measurement, Chance and Data, Space and Algebra. Each set of mathematics outcomes describes the sustained and long-term learning that is expected from students as a result of a planned program. These outcomes provide a framework for developing a mathematics curriculum that is taught to particular students in particular contexts.

Strand	Mathematics Outcomes	
Working Mathematically	**Mathematical Strategies** Call on a repertoire of general problem-solving techniques, appropriate technology and personal and collaborative management strategies when working mathematically.	**Apply and Verify** Choose mathematical ideas and tools to fit the constraints in a practical situation, interpret and make sense of the results within the context and evaluate the appropriateness of the methods used.
Number	**Understand Numbers** Read, write and understand the meaning, order and relative magnitudes of numbers, moving flexibly between equivalent forms.	**Understand Operations** Understand the meaning, use and connections between addition, multiplication, subtraction and division.
Measurement	**Understand Units and Direct Measure** Decide what needs to be measured and carry out measurements of length, capacity/volume, mass, area, time and angle to needed levels of accuracy.	**Indirect Measure** Select, interpret and combine measurements, measurement relationships and formulae to determine other measures indirectly.
Chance and Data	**Understand Chance** Understand and use the everyday language of chance and make statements about how likely it is that an event will occur based on experience, experiments and analysis.	**Collect and Process Data** Plan and undertake data collection and organise, summarise and represent data for effective and valid interpretation and communication.
Space	**Represent Spatial Ideas** Visualise, draw and model shapes, locations and arrangements and predict and show the effect of transformations on them.	**Reason Geometrically** Reason about shapes, transformations and arrangements to solve problems and justify solutions.
Algebra	**Functions** Recognise and describe the nature of variation in situations, interpreting and using verbal, symbolic, tabular and graphical ways of representing variation.	**Express Generalities** Read, write and understand the meaning of symbolic expressions, moving flexibly between equivalent expressions.

Working Mathematically

The information and approaches to teaching and learning described in the Resource Books incorporate a Working Mathematically approach.

Algebra

This series does not include a Resource Book for Algebra because students do not typically achieve the outcomes for Algebra while at primary school.

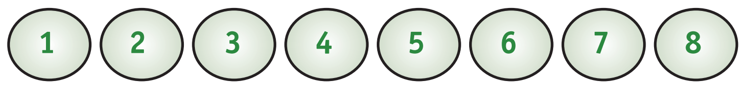
	Reason Mathematically Investigate, generalise and reason about patterns in number, space and data, explaining and justifying conclusions reached.
	Calculate Choose and use a repertoire of mental, paper and calculator computational strategies for each operation, meeting needed degrees of accuracy and judging the reasonableness of results.
	Estimate Make sensible direct and indirect estimates of quantities and are alert to the reasonableness of measurements and results.
	Interpret Data Locate, interpret, analyse and draw conclusions from data, taking into account data collection techniques and chance processes involved.
	Equivalence Equations and Inequalities Write equations and inequalities to describe the constraints in situations, and choose and use appropriate solution strategies, interpreting solutions in the original context.

Levels of Achievement

For each mathematics outcome, there are eight Levels of Achievement. The ***First Steps in Mathematics*** Resource Books focus on Levels 1 to 5 of these statements because they cover the typical range of achievement in primary school.

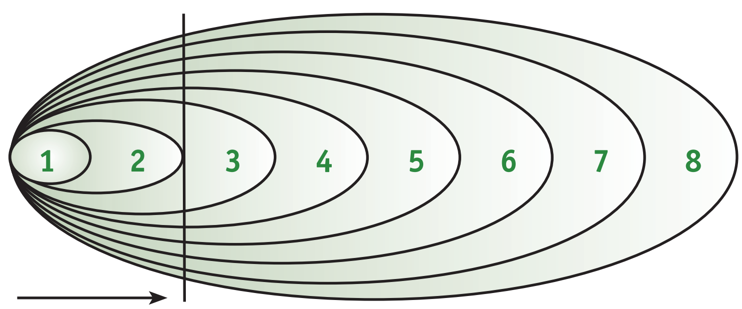
The Levels of Achievement describe markers of progress towards full achievement of the outcome. Each student's achievement in mathematics can be monitored and success judged against the Levels of Achievement.

The Levels of Achievement are developmental and cumulative in the sense that the learning described at each level encompasses all the learning described in previous levels. They also provide information about the learning required for achievement of the next level. The Levels of Achievement are not meant to be read in isolation as if they were a set of distinct outcomes to be dealt with one after the other like this:



Students do not complete one level before they begin working on the next. Rather, while students are working towards achieving a particular level, they will also be working towards achieving the subsequent levels, as the following diagram illustrates.

Outcome Level



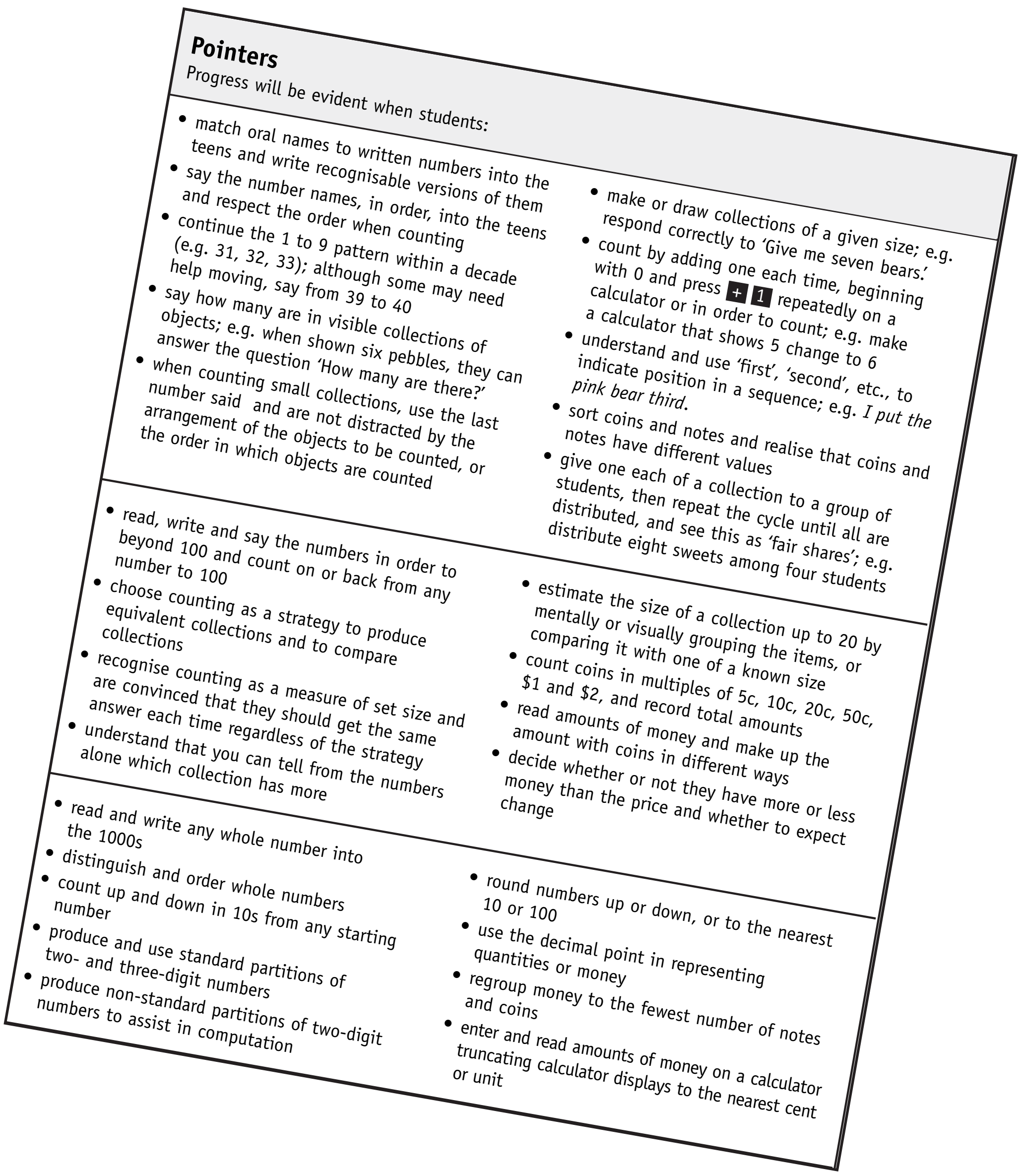
This approach has important implications for curriculum planning. While achieving Levels 1 and 2, students should already be learning many of the things needed to achieve Level 3, as well as some of the things that will lead to achieving the later levels.

Since students achieve the outcomes at different rates, it is anticipated that students in the same class will be working at different levels. The levels students are working to achieve will depend on their previous learning experiences, the emphasis of the curriculum, and a range of personal variables. Personal variables could include students' abilities, interests and experiences beyond school.

Pointers

Each Level of Achievement has a series of Pointers. They provide examples of what students might typically do if they have achieved a level. The Pointers help clarify the meaning of the mathematics outcome and the differences between the Levels of Achievement.

Note that the Pointers are neither comprehensive nor exhaustive. They are meant to serve as a guide only. They are not intended to be used as a checklist for students' achievements or the learning experiences they need at each level.



Pointers
Progress will be evident when students:

- match oral names to written numbers into the teens and write recognisable versions of them
- say the number names, in order, into the teens and respect the order when counting
- continue the 1 to 9 pattern within a decade (e.g. 31, 32, 33); although some may need help moving, say from 39 to 40
- say how many are in visible collections of objects; e.g. when shown six pebbles, they can answer the question 'How many are there?'
- when counting small collections, use the last number said and are not distracted by the arrangement of the objects to be counted, or the order in which objects are counted
- make or draw collections of a given size; e.g. respond correctly to 'Give me seven bears.'
- count by adding one each time, beginning with 0 and press + 1 repeatedly on a calculator or in order to count; e.g. make a calculator that shows 5 change to 6
- understand and use 'first', 'second', etc., to indicate position in a sequence; e.g. *I put the pink bear third.*
- sort coins and notes and realise that coins and notes have different values
- give one each of a collection to a group of students, then repeat the cycle until all are distributed, and see this as 'fair shares'; e.g. distribute eight sweets among four students

- read, write and say the numbers in order to beyond 100 and count on or back from any number to 100
- choose counting as a strategy to produce equivalent collections and to compare collections
- recognise counting as a measure of set size and are convinced that they should get the same answer each time regardless of the strategy
- understand that you can tell from the numbers alone which collection has more
- estimate the size of a collection up to 20 by mentally or visually grouping the items, or comparing it with one of a known size
- count coins in multiples of 5c, 10c, 20c, 50c, $1 and $2, and record total amounts
- read amounts of money and make up the amount with coins in different ways
- decide whether or not they have more or less money than the price and whether to expect change

- read and write any whole number into the 1000s
- distinguish and order whole numbers
- count up and down in 10s from any starting number
- produce and use standard partitions of two- and three-digit numbers
- produce non-standard partitions of two-digit numbers to assist in computation
- round numbers up or down, or to the nearest 10 or 100
- use the decimal point in representing quantities or money
- regroup money to the fewest number of notes and coins
- enter and read amounts of money on a calculator truncating calculator displays to the nearest cent or unit

Key Understandings

The Key Understandings are the cornerstone of the ***First Steps in Mathematics*** Resource Books. The Key Understandings:

- describe the mathematical ideas, or concepts, which students need to know in order to achieve the outcome
- explore how these mathematical ideas relate to the levels of achievement for the mathematics outcomes
- suggest what inputs, or experiences, teachers should plan for students so they achieve the outcome
- provide a basis for the recognition and assessment of what students already know and still need to know in order to progress
- indicate the emphasis of the curriculum at particular stages
- provide content and pedagogic advice to assist with planning the curriculum at the classroom and whole-school levels.

It is important that the mathematical ideas described in each Key Understanding are drawn out of the classroom activities and made explicit for students. Students should use their own informal language to talk about the ideas contained in the Key Understandings. They should not be expected to learn the words of the Key Understandings.

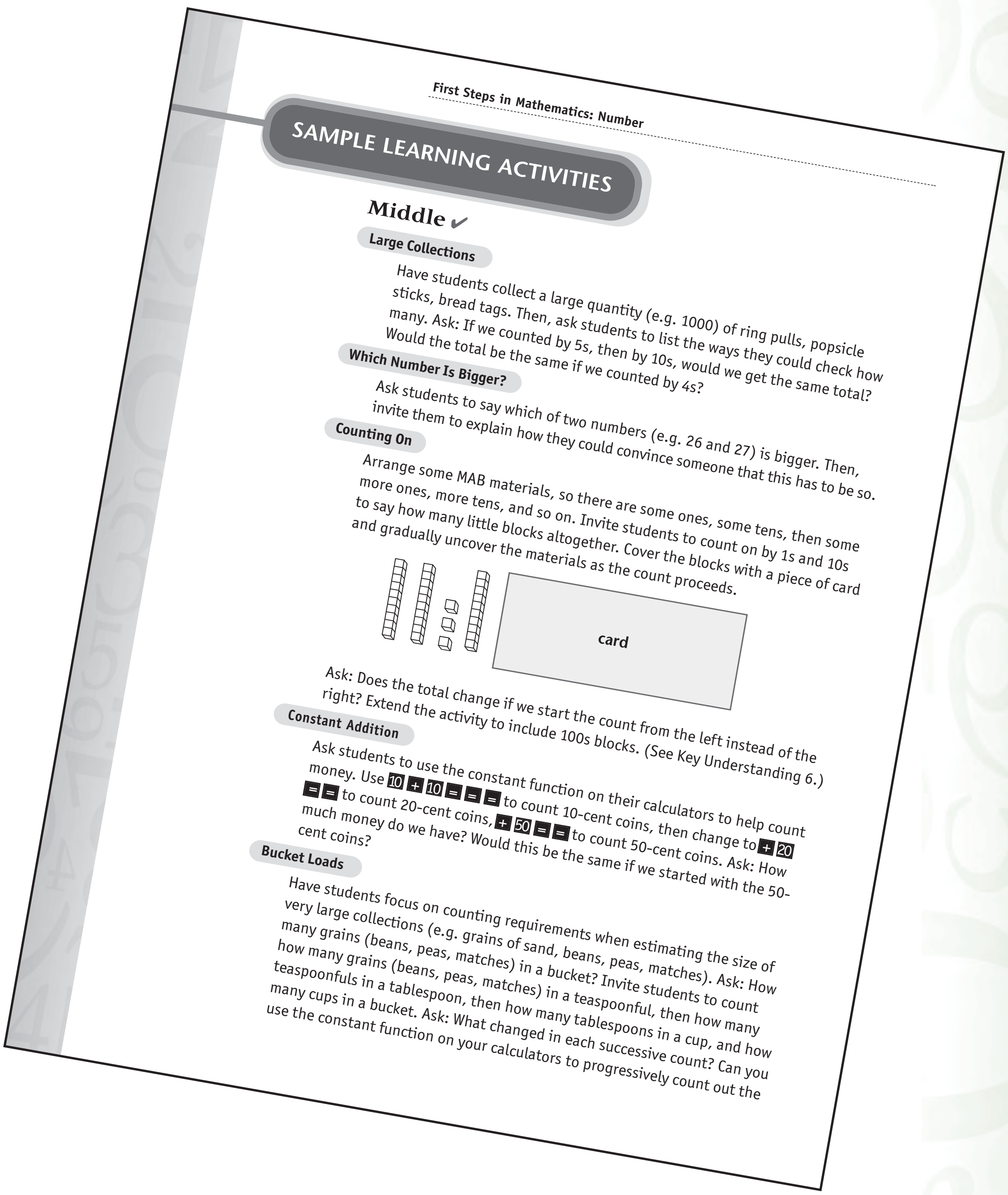
The number of Key Understandings for each mathematics outcome varies according to the number of 'big mathematical ideas' students need to achieve each outcome.

Sample Learning Activities

For each Key Understanding, there is a range of Sample Learning Activities that teachers could use to develop the mathematical idea (or Key Understanding). The activities are organised into three broad groups.

- Beginning activities are suitable for Kindergarten to Year 3 students.
- Middle activities cater for Year 3 to Year 5 students.
- Later activities are designed for Year 5 to Year 7 students.

If students in the later years have not had sufficient prior experience, teachers may need to select and adapt activities from earlier groups.



First Steps in Mathematics: Number

SAMPLE LEARNING ACTIVITIES

Middle ✔

Large Collections

Have students collect a large quantity (e.g. 1000) of ring pulls, popsicle sticks, bread tags. Then, ask students to list the ways they could check how many. Ask: *If we counted by 5s, then by 10s, would we get the same total? Would the total be the same if we counted by 4s?*

Which Number Is Bigger?

Ask students to say which of two numbers (e.g. 26 and 27) is bigger. Then, invite them to explain how they could convince someone that this has to be so.

Counting On

Arrange some MAB materials, so there are some ones, some tens, then some more ones, more tens, and so on. Invite students to count on by 1s and 10s to say how many little blocks altogether. Cover the blocks with a piece of card and gradually uncover the materials as the count proceeds.

card

Ask: *Does the total change if we start the count from the left instead of the right?* Extend the activity to include 100s blocks. (See Key Understanding 6.)

Constant Addition

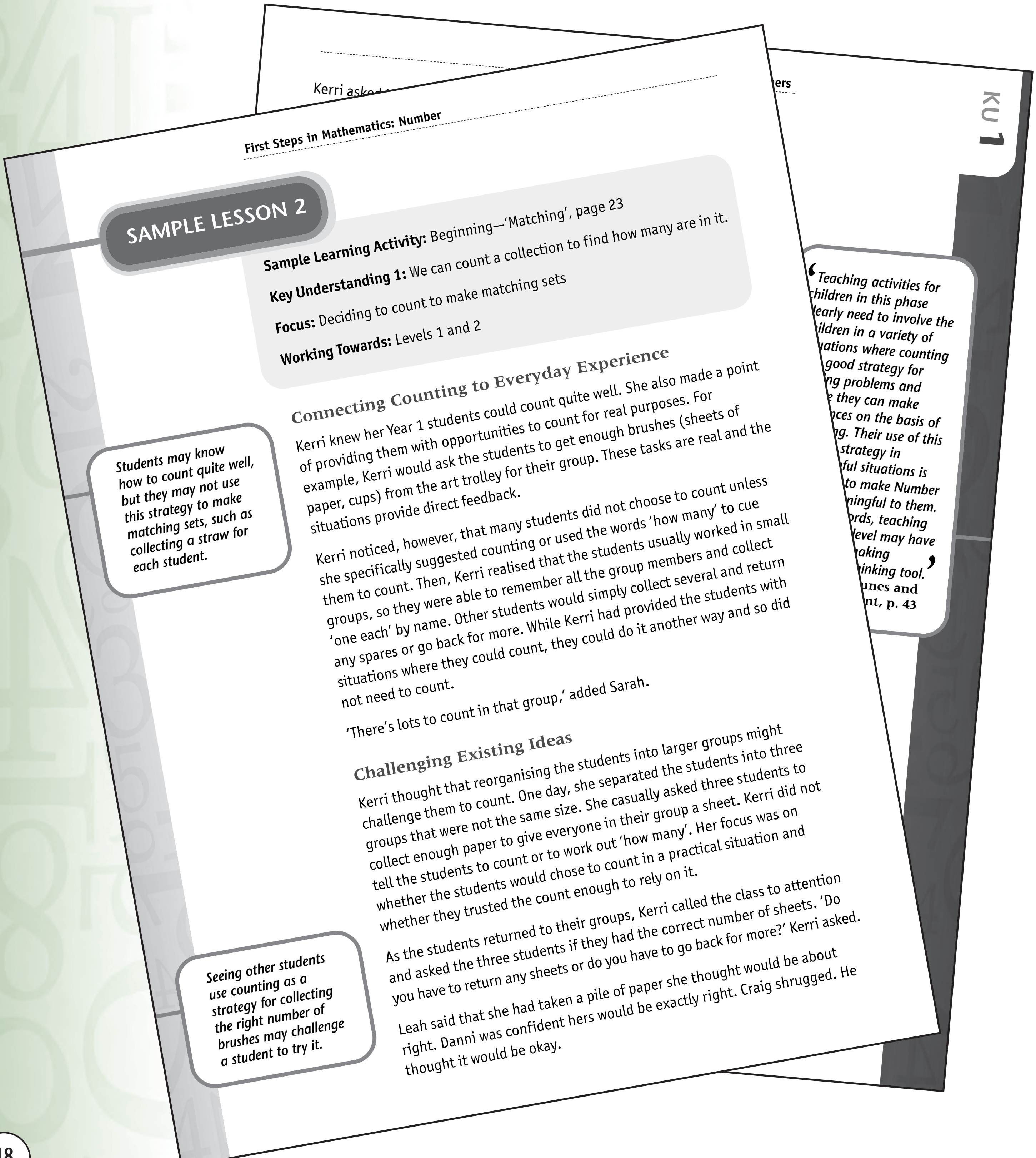
Ask students to use the constant function on their calculators to help count money. Use 10 + 10 = = = = to count 10-cent coins, then change to + 20 = = to count 20-cent coins, + 50 = = to count 50-cent coins. Ask: *How much money do we have? Would this be the same if we started with the 50-cent coins?*

Bucket Loads

Have students focus on counting requirements when estimating the size of very large collections (e.g. grains of sand, beans, peas, matches). Ask: *How many grains (beans, peas, matches) in a bucket?* Invite students to count how many grains (beans, peas, matches) in a teaspoonful, then how many teaspoonfuls in a tablespoon, then how many tablespoons in a cup, and how many cups in a bucket. Ask: *What changed in each successive count? Can you use the constant function on your calculators to progressively count out the*

Sample Lessons

The Sample Lessons illustrate some of the ways teachers can use the Sample Learning Activities for the Beginning, Middle and Later groups. The emphasis is on how teachers can focus students' attention on the mathematics of the activity.



First Steps in Mathematics: Number

SAMPLE LESSON 2

Sample Learning Activity: Beginning—'Matching', page 23

Key Understanding 1: We can count a collection to find how many are in it.

Focus: Deciding to count to make matching sets

Working Towards: Levels 1 and 2

Connecting Counting to Everyday Experience

Kerri knew her Year 1 students could count quite well. She also made a point of providing them with opportunities to count for real purposes. For example, Kerri would ask the students to get enough brushes (sheets of paper, cups) from the art trolley for their group. These tasks are real and the situations provide direct feedback.

Students may know how to count quite well, but they may not use this strategy to make matching sets, such as collecting a straw for each student.

Kerri noticed, however, that many students did not choose to count unless she specifically suggested counting or used the words 'how many' to cue them to count. Then, Kerri realised that the students usually worked in small groups, so they were able to remember all the group members and collect 'one each' by name. Other students would simply collect several and return any spares or go back for more. While Kerri had provided the students with situations where they could count, they could do it another way and so did not need to count.

'There's lots to count in that group,' added Sarah.

Challenging Existing Ideas

Kerri thought that reorganising the students into larger groups might challenge them to count. One day, she separated the students into three groups that were not the same size. She casually asked three students to collect enough paper to give everyone in their group a sheet. Kerri did not tell the students to count or to work out 'how many'. Her focus was on whether the students would chose to count in a practical situation and whether they trusted the count enough to rely on it.

As the students returned to their groups, Kerri called the class to attention and asked the three students if they had the correct number of sheets. 'Do you have to return any sheets or do you have to go back for more?' Kerri asked.

Seeing other students use counting as a strategy for collecting the right number of brushes may challenge a student to try it.

Leah said that she had taken a pile of paper she thought would be about right. Danni was confident hers would be exactly right. Craig shrugged. He thought it would be okay.

'Did You Know?' Sections

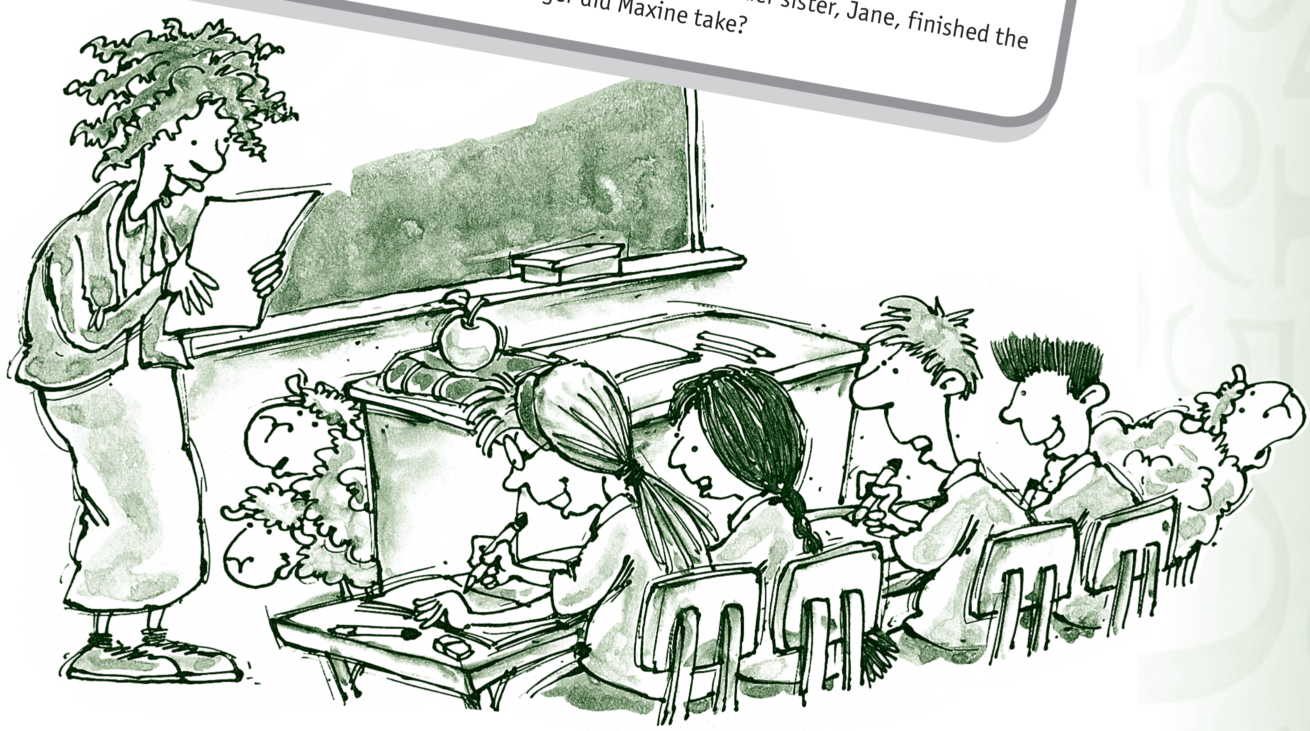
For some of the Key Understandings, there are 'Did You Know?' sections. These sections highlight common understandings and misunderstandings that students have. They also suggest diagnostic activities that teachers may wish to try with their students.

Did You Know?

Sometimes, we forget to provide real experience in deciding what operation is needed. Practice on 'word problems' might only appear on pages labelled, say, 'addition'. Or, there may be other obvious cues so that students do not really need to think much about the situation or choose the operation. Also, in daily life, there may be extraneous information and we have to choose which numbers are relevant as well as choosing the operation. Ask students to do the following activity and see what they do.

Tick the problems for which 379 – 280 would give the answer. Give reasons for your choices.

- ❑ 379 sheep had to be put through the dip. 280 sheep had been dipped by lunchtime. 80 of the sheep were black and 200 were white. How many sheep still have to be dipped?
- ❑ There was 379 kilograms of dog food stored in the factory's freezer. 280 kilograms of it was sold to local shops. How much dog food was left in the freezer?
- ❑ 379 posts are needed for the eastern boundary of the fence line and 280 posts are needed for the northern boundary. How many posts are needed altogether?
- ❑ Maxine finished the car rally in 379 minutes. Her older sister, Jane, finished the rally in 280 minutes. How much longer did Maxine take?



CHAPTER 5

Understanding the Diagnostic Maps

What Are the Diagnostic Maps?

For each set of outcomes, a Diagnostic Map is provided. The Diagnostic Maps describe characteristic phases in the development of students' thinking about the major mathematical concepts of each set of outcomes. The maps help teachers:

- understand why students seem to be able to do some things and not others
- realise why some students may be experiencing difficulties while others are not
- indicate the challenges students need to move their thinking forward, to refine their preconceptions, overcome any misconceptions, and so achieve the outcomes
- interpret their students' responses to activities.

Each map includes key indications of students' understanding and growth, and the consequences of this growth on their learning. This information is crucial for teachers making judgments about their students' level of understanding of mathematics. It enhances teachers' judgments about what to teach, to whom and when to teach it.

How to Read the Diagnostic Maps

The Diagnostic Map for Measurement has five phases. The diagram on pages 22 to 23 shows the first two phases of the Diagnostic Map for Measurement:

- Emergent
- Matching and Comparing.

There are three more phases: Quantifying, Measuring and Relating. The text in each shaded section describes students' major preoccupations, or focus, during that phase of thinking about Measurement.

The 'By the end' section of each phase provides examples of what students typically think and are able to do as a result of having worked through the phase.

The achievements described in the 'By the end' section should be read in conjunction with the 'As students move from' section. Together, these two sections illustrate that although students might have developed a range of important understandings as they passed through the phase, they might also have developed some unconventional or unhelpful ideas. Both of these sections of the Diagnostic Map are intended as a useful guide only. Teachers will recognise more examples of similar thinking in the classroom.

Diagnostic Map: Measurement

During the Emergent Phase

Initially, students attend to overall appearance of size recognising one thing as perceptually bigger than another and using comparative language in a fairly undifferentiated way (big/small) rather than as describing comparative size (bigger/smaller). Over time, they note that their communities distinguish between different forms of bigness (or size) and make relative judgments of size.

As a result, students begin to understand and use the everyday language of attributes and comparison used within their home and school environments, differentiating between attributes that are obviously perceptually different.

This part of the Diagnostic Map describes students' major preoccupations during the phase.

By the end of the Emergent phase, students typically:

- distinguish tallness from heaviness, fatness and how much things hold
- start to distinguish different forms of length and to use common contextual distractions; e.g. distinguish wide from tall
- use differentiated bi-polar pairs to describe things; e.g. thin-fat, heavy-light, tall-short
- describe two or three obvious measurement attributes of the same thing; e.g. tall, thin and heavy
- describe something as having more or less than something else, as taller or fatter.

This part of the Diagnostic Map shows what students know or can do as a result of having made the major conceptual shift of that phase.

During the Matching and Comparing phase

Students match in a conscious way to decide which is bigger by familiar, readily perceived and distinguished attributes such as length, mass, capacity and time. They also repeat copies of objects, amounts and actions to decide: 'How many fit (balance, match) a provided object or event?'

As a result, students learn to directly compare things to decide which is longer, fatter, heavier, holds more, or took longer. They also learn what people expect them to do in response to questions (How long? How tall? How wide or heavy? How much time? How much does it hold?), or when explicitly asked to measure something.

By the end of the Matching and Comparing phase, students typically:

- attempt to focus on a particular attribute to compare two things; e.g. how much the jar holds
- know that several things may be in different orders when compared by different attributes
- line up the base of two sticks when comparing their lengths and fit regions on top of each other to compare area
- use the everyday notion of 'how many fit' and count how many repeats of an object fit into or match another; e.g. How many pens fit along the table? How many potato prints cover the sheet? How many blocks fit in the box?
- count units and call it 'measuring'; e.g. *I measured and found the jar holds a bit more than seven scoops.*
- use 'between' to describe measurements of uni-dimensional quantities (height, mass, capacity, time); e.g. *It weighs between seven and eight marbles.*
- refer informally to part units when measuring uni-dimensional quantities; e.g. *Our room is six-and-a-bit metres long.*

Most students will enter the Matching and Comparing phase between 3 and 5 years of age.

As students move from the Emergent phase to the Matching and Comparing phase, they:

- may not 'conserve' measures; e.g. thinking that moving a rod changes its length, pouring changes 'how much', cutting up paper makes more surface
- may visually compare the size of two things but make no effort to match; e.g. say which stick is longer without lining up the bases, or which sheet of paper is bigger without superimposing
- compare time spans but may not take into account different starting times; e.g. deciding that the TV program which finished last was on the longest
- use bi-polar pairs but may have difficulty with some comparative terms; e.g. heft to decide which is heavier but say both are heavy because both hands go down
- may distinguish two attributes (such as tallness and weight) but not understand that the two attributes may lead to different orders of size for a collection, expecting the order for tallness and the order for weight to be the same
- when describing different attributes of the same thing (tall, thin, heavy), many will be confused by a request to compare two things by different attributes, particularly if the comparisons lead to different orders
- often do not think to use counting to say how big or how much bigger; e.g. they may 'weigh' something by putting it into one side of a balance and smaller objects into the other side but do not count the objects.

These are the learning challenges for the Matching and Comparing phase.

This part of the Diagnostic Map shows the learning challenges for the next phase.

Most students will enter the Quantifying phase between 5 and 6 or more years of age.

As students move from the Matching and Comparing phase, they:

- know that while ordering objects by different attributes might lead to different orders, they may still be influenced by the more dominant perceptual features; e.g. they may think the tallest container holds the most
- may 'count units' in order to compare two things but be fairly casual in their repetitions of units, not noticing gaps or overlaps; e.g. placing the first 'unit' away from the end when measuring how much a container holds; not stopping their claps immediately when the music stops
- do not necessarily expect the same 'answer' each time when deciding how many fit
- may not think to use unit information to answer questions such as: Which holds more? Will the table slide though the doorway?
- may not see the significance of using a common unit to compare two things and when using different units, let the resulting number override their perceptual judgments
- while many will have learned to use the centimetre marks on a conventional rule to 'measure' lengths, they often do not see the connection between this process and the repetition of units.

These are the learning challenges for the Quantifying phase.

There are three more phases.

How Do Students Progress Through the Phases?

Students who have passed through one phase of the Diagnostic Map are entering the next phase. They bring behaviours and understandings from one phase to the next. For example, the text next to the 'During the Emergent Phase', 'By the end of...' section describes the behaviours students bring to the Matching and Comparing phase. The 'As students move from...' section includes the preconceptions, partial conceptions and misconceptions that students may have developed along the way. These provide the learning challenges for the next phase.

Linking the Diagnostic Maps and Levels of Achievement

Students are unlikely to achieve certain levels of the outcomes unless they have moved through certain phases of the Diagnostic Map. However, passing through the phase does not guarantee that the level of the outcome has been achieved. Students might have the conceptual development necessary for achieving certain levels, but without access to a curriculum that enables them to learn the necessary concepts described in the level, they will be unable to do so.

The phases help teachers interpret students' responses in terms of pre- and partial conceptions. If, for example, a student cannot count on, despite a teacher spending considerable time with that student, then the phases can help explain what the problem is. In this case, the student may not be through the Quantifying phase for Number and so may not trust the count. No amount of practice or telling the student to 'hold the number in your head' will help. The source of the problem is that the student does not trust that the initial quantity remains the same. This concept must be developed before the student can learn to count on.

How Will Teachers Use the Diagnostic Maps?

The Diagnostic Maps are intended to assist teachers as they plan for mathematics teaching and learning. The descriptions of the phases help teachers make judgments about students' understandings of the mathematical concepts. The maps will help teachers understand why students can do some things and not others, and why some students may be having difficulty achieving certain outcomes.

Initially, teachers may use the Diagnostic Maps to extend their own knowledge about how students typically learn mathematics. Knowing about the major conceptual shifts in each phase and their links to the Levels of Achievement will help teachers decide which Key Understandings should be the major focus for classroom planning.

Familiarity with the behaviours described in the phases will enhance the judgments teachers make about what they observe students doing and saying during lessons. The information obtained over time about the major preoccupations of students informs ongoing planning. As teachers begin to understand the typical behaviours of each phase, this planning process will become more efficient.

Using the Diagnostic Maps: Case Story

The following case story describes how one teacher, Jenny, used the Diagnostic Map for Measurement to interpret her students' responses to some of the Sample Learning Activities intended to help them learn about units.

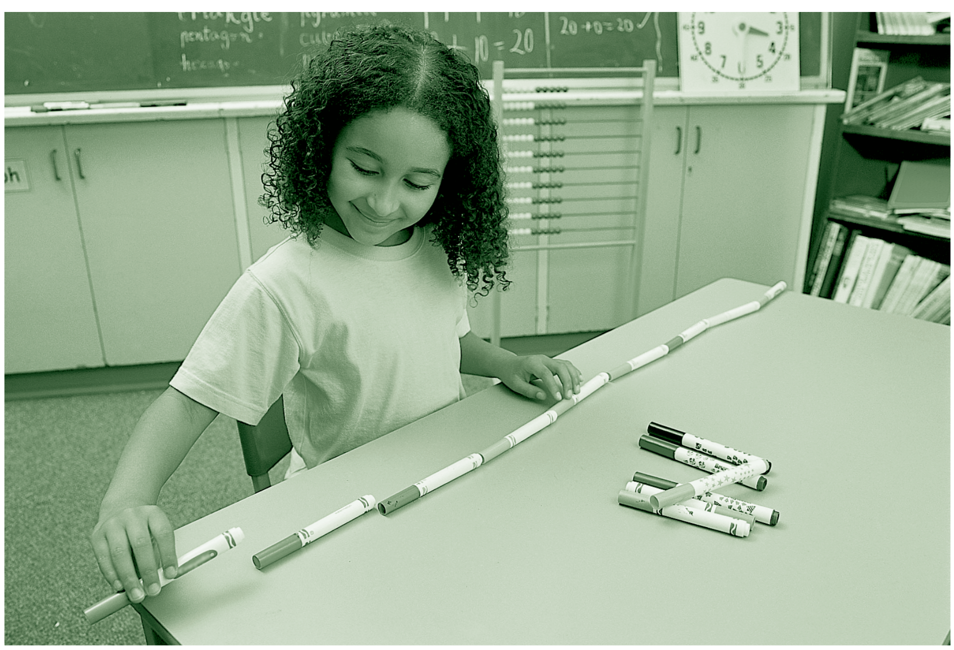
Note: The photos in this section are generic in nature and not specific to the case story recorded here.

Jenny knew that many students in her class were progressing towards the achievement of Level 2 of the outcomes for Understand Units and Direct Measurement. The activities she selected from the Middle stage of the Sample Learning Activities in Understand Units, Key Understanding 3, focused on developing students' understanding of units and how they are used to measure length.

Jenny observed her students as they worked in groups on a variety of activities in different parts of the classroom. Millie's group was 'measuring' the width of their desks with pens. Jenny used the questions suggested in the Sample Learning Activities to draw out the mathematical ideas she wanted them to learn.

Later, Jenny used the description of the Matching and Comparing and the Quantifying phases in the Diagnostic Map for Measurement to interpret what she observed.

Millie and her friends were asked, 'How wide is the desk?' Millie placed pens end-to-end along the edge of her desk, so she could 'measure it'. However, she did not count the pens.



Millie uses pens to measure the length of the desk.

Jenny: *Millie knows what to do in response to the question: 'How wide is the desk?' But she didn't think to count the pens.*

Jenny then asked the students whether they thought the desk could slide through the doorway. Millie didn't think to use the pens. Instead, after some thought, she eventually suggested that she could 'measure it to find out.'

Millie then chose one-centimetre cubes and placed them along the desk. She placed the cubes close together at first, but then moved them further and further apart until she reached the other side of the desk.

Millie uses one-centimetre cubes to measure the desk.

Millie did not think to count the blocks in order to say how big the desk was. She was unable to say whether the desk would fit through the doorway. Millie chose an inappropriate unit of measurement. She should have chosen something that was long like a straw or a pen. Millie is in the Matching and Comparing phase.

Jenny: *While Millie needs to learn to choose appropriate units, she needs to learn to count units first and to understand that doing this will tell her how big something is. I need to plan Sample Learning Activities from Understand Units, Key Understandings 3 and 4, that focus on these ideas. Millie needs to understand the ideas from these Key Understandings in order to achieve Level 2 of Understand Units and Direct Measure.*

The other members of Millie's group 'measured' the width of the desk with pens. Taylor then measured the width of the doorway by fitting pens across it. He decided the desk would fit because of the size of the gap left when the connected pens from the desk were fitted across the doorway.

Taylor uses pens from the desk to measure the width of the doorway.

Taylor is probably through the Matching and Comparing phase. He 'consciously matched in order to directly compare'. Taylor is matching the length formed by the connected pens with the width of the desk. Jenny has observed Taylor approach similar tasks in the same way. With this thinking, he cannot achieve Level 2 because he is directly comparing and not using units.

Learning to use units to compare the size of things is the major preoccupation of the Quantifying phase. Taylor can use direct comparison to compare lengths, but he also needs to learn how to use units.

Jenny: *I need to plan more activities from Understand Units, Key Understanding 3, to help Taylor 'use uniform units carefully to measure length' as required for the achievement of Level 2 of the Measurement outcome.*

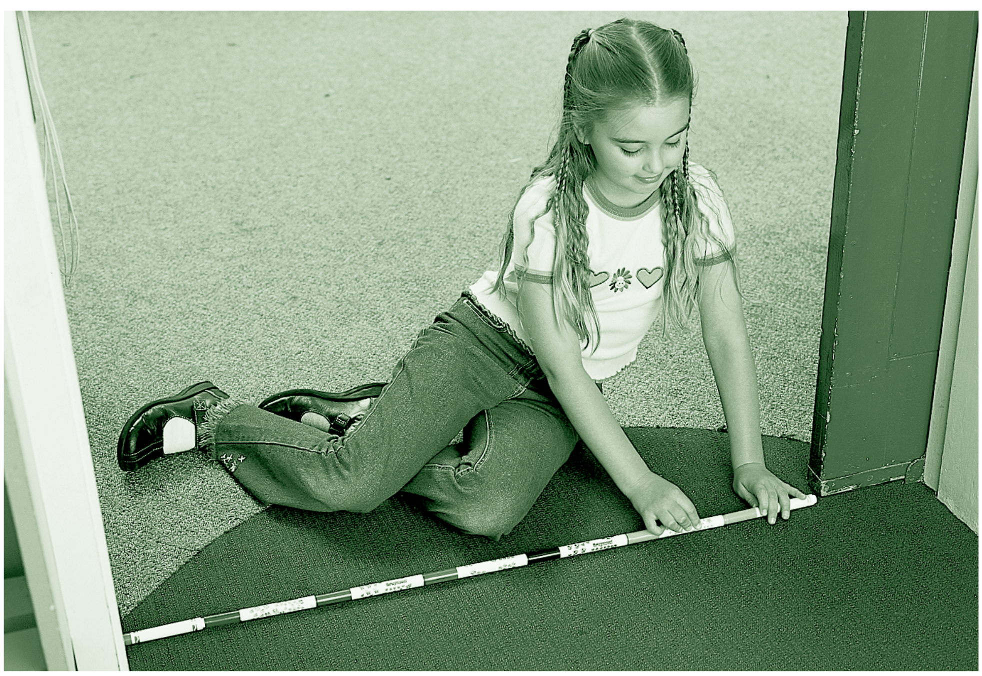
Rhiannon counted the number of straws that fit across the desk. Next, she tried to fit straws across the doorway in mid-air, then estimated the size of the 'gap' at the end. Rhiannon based her answer on visual comparison rather than the information obtained by counting with the straws. She did not use the number of straws to help her compare the widths of the desk and the doorway.

Like Taylor, Rhiannon is probably through the Matching and Comparing phase. Although, as Rhiannon has not used the count, it is hard to say whether she understands that 'counting units gives a measure of the size of the object'.

Jenny: *The Sample Learning Activities that I plan to use to help Taylor and Millie achieve Level 2 of the Measurement outcome should also be appropriate for Rhiannon.*

Rhiannon uses straws to measure the width of the doorway.

Alice compared the doorway measurement of pens with the desk measurement of five and a half pens. She decided that because the doorway fitted more pens it must be bigger. Alice appears to be through the Quantifying phase.



Alice places pens on the floor to measure the width of the doorway.

Jenny: *This suggests that Alice is entering the Measuring phase. Alice is probably in a position to achieve Level 3. She understands that the count of actual or imagined repetitions of units gives an indication of size, and that it enables two things to be compared without directly measuring them. However, to achieve Level 3, Alice will need to be able to use units to measure the range of attributes, and also use standard units for length and time. I need to focus Alice on Understand Units, Key Understandings 3, 4 and 7, and Direct Measure, Key Understandings 3, 4 and 5.*

Jenny observed and interacted with other groups of students during the week. She judged that although students responded to the Sample Learning Activities differently, most were fixed on the idea of 'how many fit?' They were through the Matching and Comparing phase and in the Quantifying phase. Her students understood that the count was related to size, but they did not use this information to compare the size of the objects. They had a partial understanding of the concept of a unit.

Jenny decided that unless she explicitly drew students' attention to this idea, they were at risk of not achieving Levels 2 and 3 of the outcome. She used the Sample Learning Activities in Understand Units, Key Understanding 3, to draw out the idea that the count described the size of the object 'being measured'. Her teaching focused on developing the thinking described in the 'During the Quantifying phase' section of the Diagnostic Map.

Considering students' responses collectively also helped Jenny see that apparently different errors may all be a result of, or indicators of, the same thinking. Planning to deal with the underlying thinking is likely to be much more efficient than trying to correct each error individually and in isolation.

CHAPTER 6

Planning with *First Steps in Mathematics*

Using Professional Judgment to Plan

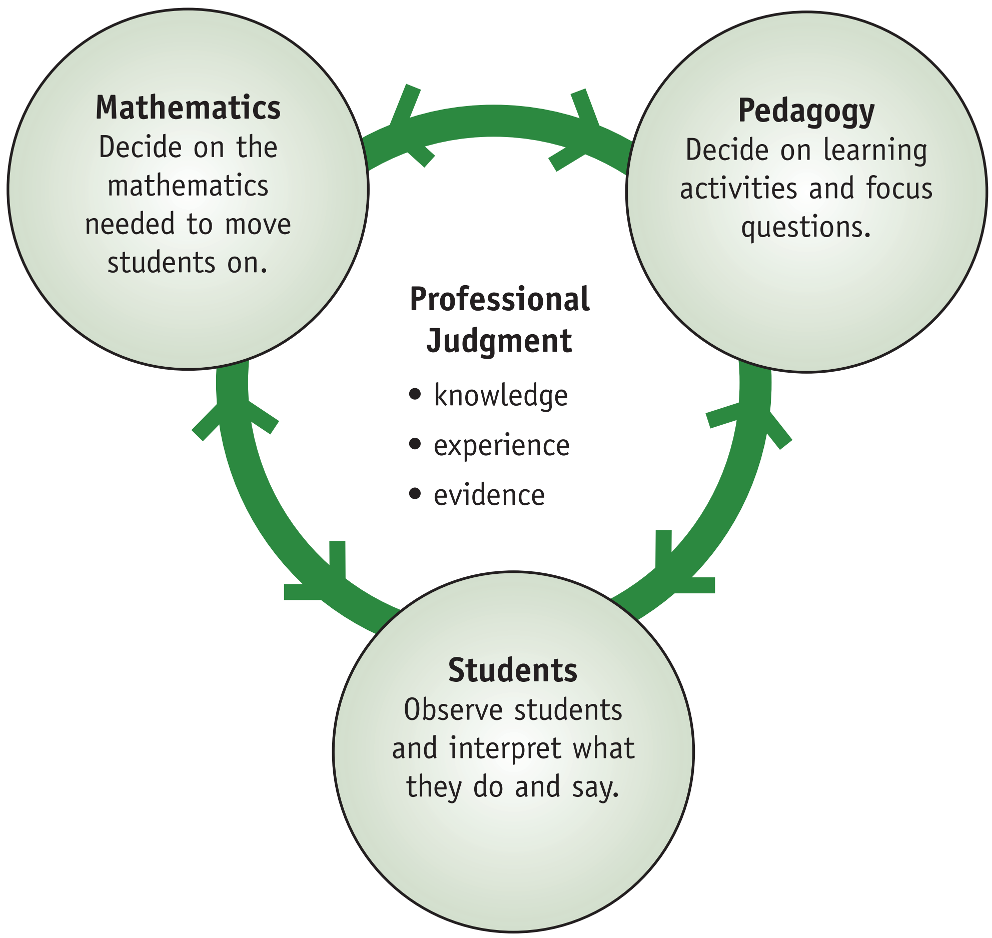
The ***First Steps in Mathematics*** Resource Books and professional development support the belief that teachers are in the best position to make decisions about how to help their students achieve the mathematics outcomes. Teachers will base these decisions on knowledge, experience and evidence.

The process of using professional judgments to plan classroom experiences for students is fluid, is dependent on the situation and context, and varies from teacher to teacher. The selection of learning activities and appropriate focus questions will be driven by each teacher's knowledge of his or her students and their learning needs, the mathematics, and mathematics-related pedagogy. The ***First Steps in Mathematics*** Resource Books and professional development focus on developing this 'content-pedagogic' knowledge.

The diagram on page 32 illustrates how these components combine to inform professional judgments. There is no correct place to start or finish, or process to go through. Circumstances and experience will determine both the starting point and which component takes precedence at any given time.

Different teachers working with different students may make different decisions about what to teach, to whom, when and how.

The process is about selecting activities that enable all students to learn the mathematics described in the outcomes. More often than not, teachers' choice of activities and focus questions will be driven by their knowledge of their students and the mathematics. At other times, teachers might select an activity to help them find out about students' existing knowledge or because of the specific mathematics in the task. Whatever the starting point, the ***First Steps in Mathematics*** Resource Books and professional development will help teachers to ensure that their mathematics pedagogy is well informed.



The examples on the opposite page show some of the different ways teachers can begin planning using ***First Steps in Mathematics***.

Focusing on the Mathematics

Teachers may choose to focus on the mathematics, deciding on the mathematics they think they need to move students on.

What mathematics do my students need to know?

Mathematics
Decide on the mathematics needed to move students on.

What sections of ***First Steps in Mathematics*** do I look at?

- mathematics outcomes
- Levels of Achievement and Pointers
- Key Understandings and Key Understandings descriptions

Understanding What Students Already Know

Teachers may choose to start by finding out what mathematics their students already know.

What do my students know about these mathematics concepts?

Students
Observe students and interpret what they do and say.

What sections of ***First Steps in Mathematics*** do I look at?

- mathematics outcomes
- Levels of Achievement and Pointers
- Key Understandings and Key Understandings descriptions
- 'Did You Know?' sections
- Diagnostic Maps

Developing Students' Knowledge

Teachers may begin by planning and implementing some activities to develop their knowledge of students' learning.

What activities will help my students develop these ideas? How will I draw out the mathematical ideas from the learning activity?

Pedagogy
Decide on learning activities and focus questions.

What sections of ***First Steps in Mathematics*** do I look at?

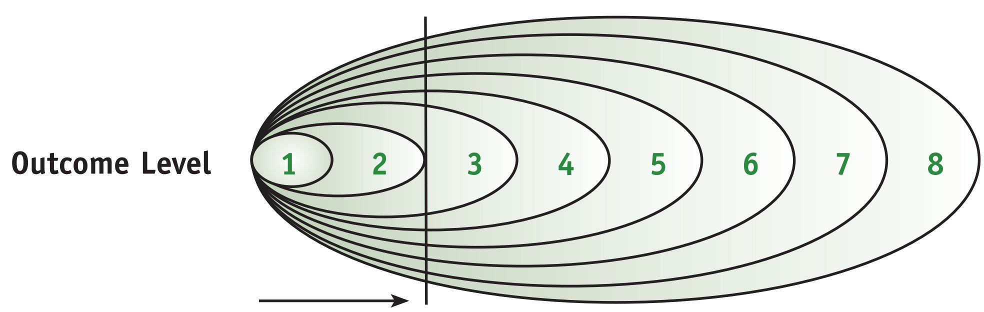
- Sample Learning Activities
- Sample Lessons
- Key Understandings and Key Understandings descriptions

Planning

The mathematics outcomes and Levels of Achievement help teachers to know where students have come from and where they are heading. This has implications for planning. While day-to-day planning may focus on the mathematics of particular Key Understandings, teachers must keep in mind the learning required for progression through the Levels of Achievement.

If a student has achieved Level 1 of Understand Units, then the majority of experiences the teacher provides will relate to achieving Level 2 of the outcome. However, some activities will also be needed that, although unnecessary for achieving Level 2, will lay important groundwork for the achievement of Level 3 and even Level 4.

For example, students do not typically understand the significance of uniform units until the middle years of primary school. Therefore, selecting uniform units is not expected for achieving Level 2 of the outcome, but it is for the achievement of Level 3. Given access to an appropriate curriculum in Measurement, most students should be able to achieve Level 3, selecting appropriate and uniform units, by the end of the middle years of primary school. If students are to develop these ideas in a timely manner, then they cannot be left until after Level 2 is achieved.



There are a number of reasons for this approach. First, it is anticipated that a considerable number of students will enter the middle years of primary school having achieved Level 3 of the outcome. Second, if teachers were to wait until the middle years to start teaching about uniform units, then it is unlikely that students would develop all the necessary concepts and skills in one year.

Third, work in the middle years of primary school should not only focus on Level 3, but also provide the groundwork for students to achieve Level 4 of the outcome in the next year or two, and Level 5 some time later.

Teachers, who plan on the basis of progressive levels of achievement of the outcomes, would think about the expected long-term learning in the early years of schooling. They would provide experiences that lead to the outcomes at Level 3 and Level 4. This means students may be challenged about the significance of uniform units in the relatively simpler contexts of length and capacity during the first few years of schooling. They may not yet be ready to deal with the same ideas for other attributes. It will take several years of learning experiences across the range of attributes—mass, volume, area, time and angle—to culminate in a full understanding.

Monitoring Students over Time

By describing progressive levels of achievement that span the primary-school years, teachers can monitor students' long-term personal growth as well as their long-term progress against an external standard. This long-term monitoring is one of the reasons why a whole-school approach is essential. For example, Sarah, has achieved Level 4 for each of the Measurement outcomes while another student, Maria, has only just achieved Level 2.

By comparing Maria and Sarah's levels against the standard, their teacher is able to conclude that Sarah is progressing well, but Maria is not. This prompts Maria's teacher to investigate Maria's thinking about Measurement and to plan specific support.

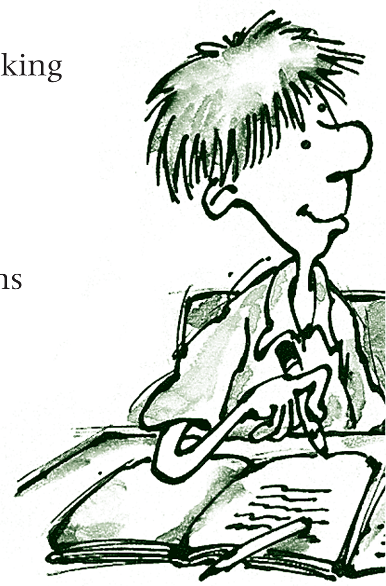
However, if two years later, Sarah has not achieved Level 5 while Maria has achieved Level 3 and is progressing well towards achieving Level 4, they would both now be considered 'on track' against an external standard. Sarah's achievement is more advanced than Maria's, but in terms of personal growth, Sarah appears to have stalled. Her progress may now be of greater concern than Maria's.

Reflecting on the Effectiveness of Planned Lessons

The fact that activities were chosen with particular outcomes in mind does not mean that they will have the desired result. Sometimes, students deal with an activity successfully, but they use different mathematics than teachers anticipated. Different activities related to the mathematics that has not been learned may need to be provided in the future.

On other occasions, what students actually learn may not be what teachers intended them to learn. Students may surprise teachers and cause them to rethink the activity.

In some instances, activities, which teachers think will help students develop particular mathematical ideas, do not generate those ideas. This can occur even when students carry out the activity correctly.



The evidence about what students are actually thinking and doing during their learning experiences is the most important source of professional learning and judgment. At the end of an activity, teachers need to ask themselves: *Have the students learned what was intended for this lesson? If not, why not?* These questions are at the heart of improving teaching and learning. Teachers make constant professional judgments about whether the implemented curriculum is resulting in the intended learning outcomes for students. If not, then teachers need to change the experiences provided.

Teachers' judgments, when planning and adjusting learning activities as they teach, are supported by a clear understanding of:

- the desired mathematics outcome of the selected activities
- what constitutes progress in mathematics
- what to look for as evidence of students' progress.

When planning day-to-day lessons, it is important for teachers to appreciate that many of the same activities will be appropriate for students who are working at a range of levels. Teachers can accommodate the differences in understanding and development among students by:

- asking different questions of individual students and groups of students
- providing extension activities for selected students
- giving particular students opportunities to do different things with the activities.

How Other Teachers Have Used the Materials: Case Stories

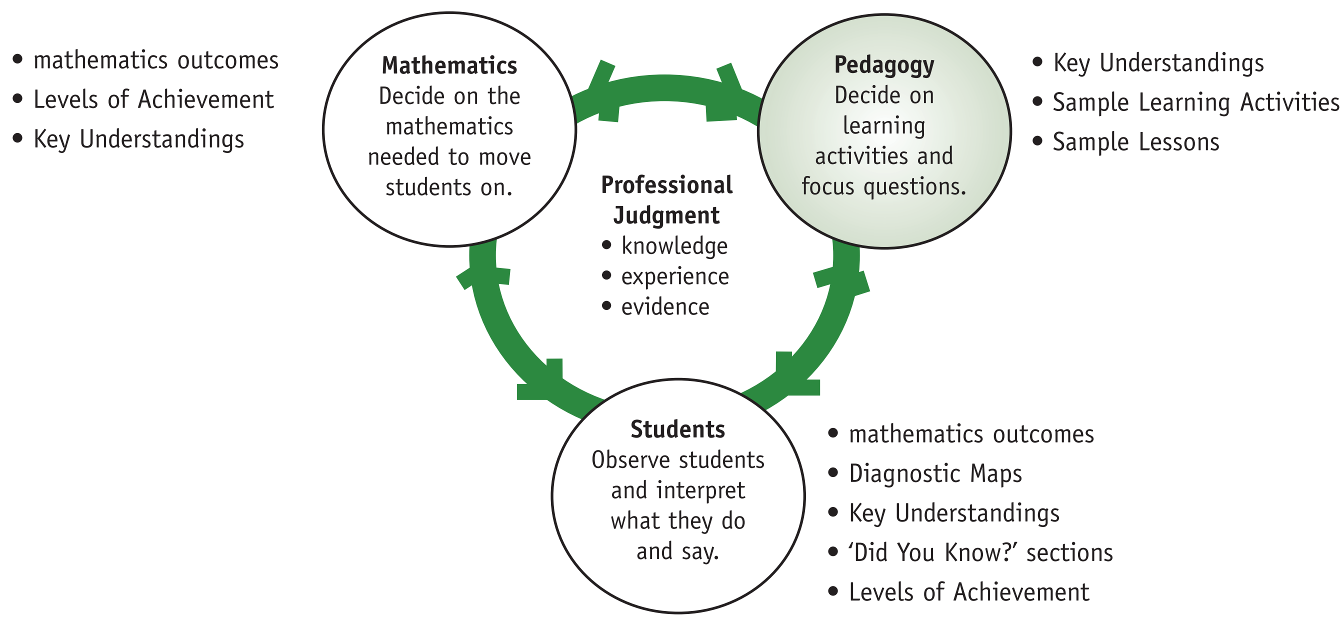
How the ***First Steps in Mathematics*** Resource Books are used will vary from teacher to teacher. On the following pages, are some descriptions provided by classroom teachers from different year levels of how they use the materials to plan with their students.

Case Story 1 — Year Level: 3

Developing Students' Knowledge

I am teaching Year 3 for the first time this year after teaching older students for a number of years. Initially, I was concerned that my teaching would be pitched too high for the younger students, so I was a bit anxious about how to get started. I decided to try some Sample Learning Activities from ***First Steps in Mathematics: Number*** to get a feel for what the students knew.

I started with Understand Numbers. The ticks on the Key Understandings overview page gave me a quick guide to what I should focus on. In my first week, I tried some activities from Key Understanding 4 about the patterns in the way we say numbers. The students really loved the activities. They could see that our mathematics lessons were going to be challenging, but enjoyable. Then, I looked more closely at the information in Key Understanding 4. It helped me to clarify the specific mathematics that my students had learned about the patterns in the way we say numbers and to decide what else I needed to teach them.



This teacher started with the pedagogy.

Case Story 2 — Year Level: 5

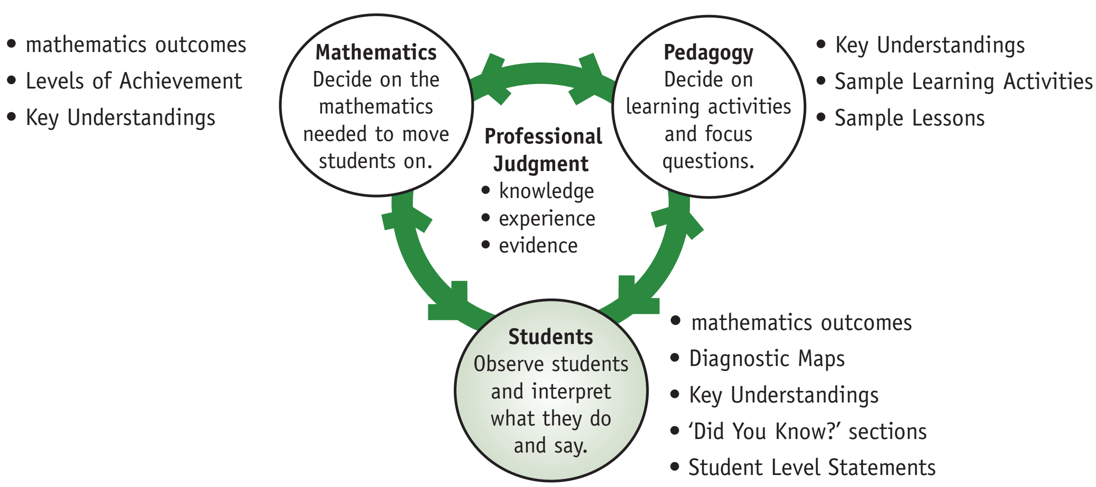
Understanding What Students Already Know

The teachers at my school met to decide which Key Understandings needed to be emphasised at each year level. The Key Understandings overview page helped us to see that the Key Understandings about the patterns in the number system supported the achievement of Levels 2 and 3 for Understand Numbers.

I used the ideas in the 'Did You Know?' sections for Understand Numbers, Key Understandings 4 and 5, to find out whether my class responded in a similar way. Next, I selected Sample Learning Activities from the Middle group for Understand Numbers, Key Understandings 4, 5 and 6. Then, I re-read the information in each Key Understanding to remind myself about the mathematics I needed to focus on. Over time, I built up a picture of what my students knew and didn't yet know in relation to the Key Understandings using the dot points from Key Understandings 4 and 5.

I used the Diagnostic Map regularly to help me interpret my students' responses. Some students, for example, were experiencing difficulty adding and subtracting two-digit numbers where re-grouping was involved. Using the map helped me to identify the likely source of the problem. Although these students were 'through' the Quantifying phase, they could not partition numbers that they could not visualise or represent as quantities. The Sample Learning Activities from Understand Numbers, Key Understandings 2 and 6, and Calculate, Key Understanding 2, will help my students develop these ideas.

I referred back to the mathematics outcome for Understand Numbers. I thought again about my students' achievements in relation to the outcome. The information in the Resource Book about what students know in relation to Levels 2 and 3 really helped me to know what to look for.



This teacher started with the students.

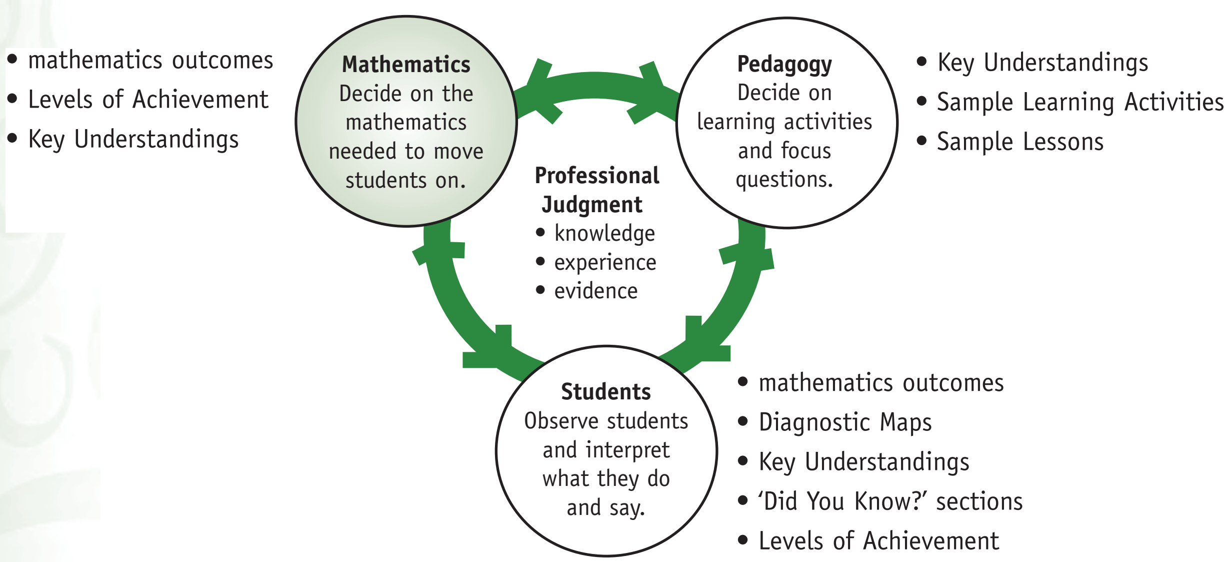
Case Story 3 Year Level: 7

Focusing on the Mathematics

I am a Year 7 classroom teacher. When planning my unit on adding and subtracting fractions, I looked through ***First Steps in Mathematics: Number*** to find a reference to the topic. I found Calculate, Key Understanding 7 ('we can calculate with fractions'), so I turned to the relevant page and read the description. I was surprised at my lack of knowledge in this area. Then, I returned to the mathematics outcome at the front of this section. Suddenly, it dawned on me what the outcome really meant and what I was trying to get my students to do.

Next, I looked at the Later Sample Learning Activities for Key Understanding 7, and decided to do some of them with my class the following week. I was surprised at what my students were saying and doing during the activities. When I read the descriptions of each phase in the Diagnostic Map for Number, I realised that most of my students probably did not know what a fraction was. I referred to the Understand Fractional Numbers section of the text, and decided to work on developing my students' Key Understandings about fractions before I spent any more time doing calculations with them.

So, in my planning, I have to jump from one section of the Resource Book to another, depending on the mathematics my students need to learn. As we progress, I keep referring to the mathematics outcomes and the Levels of Achievement to decide what mathematics I want my students to learn. Then, I choose activities from the relevant Key Understandings to help them.



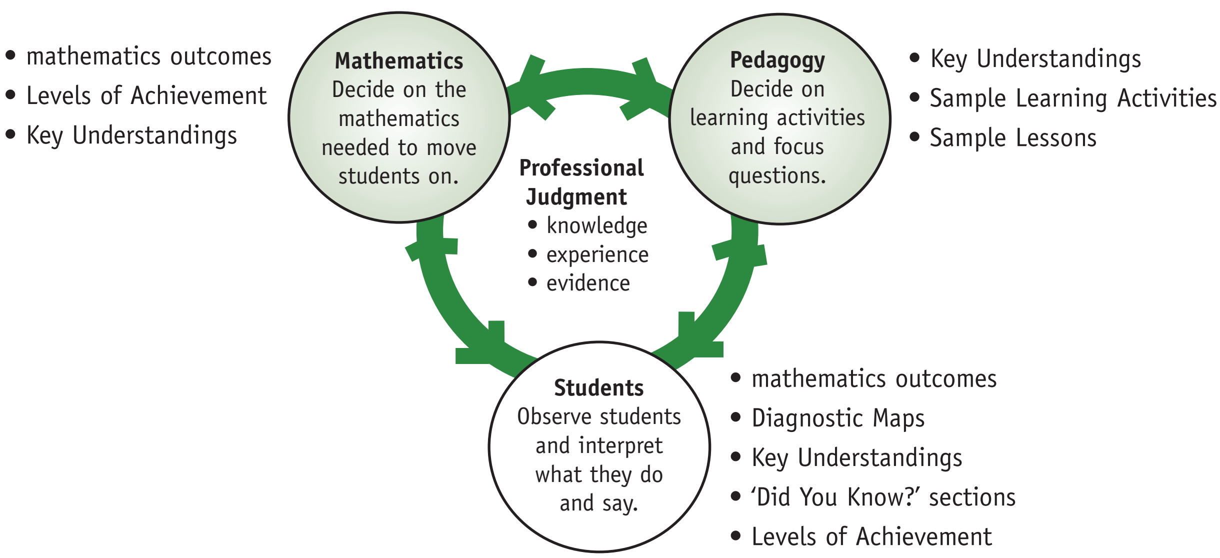
This teacher started with the mathematics.

Case Story 4 Year Level: 3

Focusing on the Mathematics and Developing Students' Knowledge

I spent the first four weeks of term 2 working on Understand Numbers, Key Understandings 1, 2 and 4, and I have seen my students make considerable progress in these understandings. Most of them can now count large collections accurately and trust the count to say how many. Most students are also able to say the sequence of numbers using the 1 to 9 sequence through the decades. However, a couple of students need to be reminded of some of these (e.g. 70 follows 69).

I thought it was time to focus on basic place-value knowledge (Key Understanding 5) to help students link the way we count numbers to the way we write them. I wrote up a plan for next week's mathematics lessons (see pages 42 to 43). I will see how my students go with the planned activities and then make further decisions about where to go to next.



This teacher started with the pedagogy and the mathematics.

Classroom Plan for Week 5, Term 2

Outcome/Key Understanding	Mathematical Focus	Activities	
Main focus: Understand Numbers, Key Understanding 5 There are patterns in the way we write whole numbers which help us to remember their order.	• The position or place of a digit tells us the quantity it represents. • Zero is used as a place-holder. • We can group collections to make it easier to see how many there are (leading from Key Understanding 2). • The quantity always remains the same however the collection is grouped; e.g. 5 groups of 10 items is exactly the same quantity as 50 individual items.	• Use the constant function on a calculator to see what happens when we add numbers in tens and hundreds. Start from non-decade numbers, e.g. 54 or 168. • Play 'Wipeout' (Sample Learning Activity, p. 14).	
Minor focus: Understand Numbers, Key Understanding 6 Place value helps us to think of the same whole number in different ways and this can be useful.	• Groupings based on tens are unique to our number system and make it very easy to count forwards and backwards in tens and hundreds.	• Modify 'Handful of Beans' activity (p. 15) to count a collection of straws by grouping in different ways. Have students work individually. • Following above activity, each student combines their collection with another student's. They count by tens to find out how many altogether. Students write their total number. • Extend the above activity for more able students. Ask: If you were going to count all of the straws in the class, how could you group them to help make the count easier?	

Year Level: 3

Focus Questions	Observations/Anecdotes
• What number changes? • What number stays the same? Why? • Why do the hundreds as well as the tens change when we keep adding 10 to a number? • Ask students to read their numbers out loud. Ask: What does the 3 in 234 mean? How do you know?	Many students were fascinated by the way the numbers changed. Most were able to say which number they would change next and why. I'm not sure that this really shows me very much though. I need to keep working on this.
• How do you think you can get rid of the 5 in 256? • Why doesn't 256 become 26 when you enter –50 in your calculator? • Why do you need to –50 instead of 5 to 'wipe out' the 5 in 256?	Ari, Kim and Jedda could not explain why it had to be 50 and not 5. Others could explain it, but I don't think they really understand. The next activity should help them.
• Which groups make it easier to count? Why? • How is grouping in tens helpful? • What do you notice about the way we write 27 and the tens bundles? • How many straws will there be if we 'unbundle' the tens?	Many students did not think to use groups of ten until after we talked about this. I think all students, except Anni and Pablo, need more activities that focus on this idea. We will work on it again next week.
• Ask students the same questions as above, then add: If you count by ones will you get the same number of straws? How do you know? • Say: Show me what this part of your number means in the straws. Point to the number students have written in the tens place.	Ari, Kim and Jedda were not able to tell me how many straws if we removed the elastic bands. They needed to recount to find out. I think this means that they are not through the Quantifying phase—they do not trust the count.
• Would groups of ten tens help? How many would be in this group? • What do you notice about the way we write 247 and the way we have bundled the straws? • How many straws will there be if we 'unbundle' them?	I used this activity with Anni and Pablo. They were able to tell me to make bundles of 100, and they knew how this related to the way the numbers were written. I will need to further extend these two.

Classroom Plan for Week ___, Term ___ Year Level: ___

Outcome/Key Understanding	Mathematical Focus	Activities	Focus Questions	Observations/Anecdotes